SSB of scale symmetry, fermion families and quintessence without the long-range force problem

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Abstract

We study a scale invariant two measures theory where a dilaton field ϕ has no explicit potentials. The scale transformations include a translation of a dilaton $\phi \rightarrow \phi + const$. The theory demonstrates a new mechanism for generation of the exponential potential: in the conformal Einstein frame (CEF), after SSB of scale invariance, the theory develops the exponential potential and, in general, non-linear kinetic term is generated as well. The scale symmetry does not allow the appearance of terms breaking the exponential shape of the potential that solves the problem of the flatness of the scalar field potential in the context of quintessential scenarios. As examples, two different possibilities for the choice of the dimensionless parameters are presented where the theory permits to get interesting cosmological results. For the first choice , the theory has standard scaling solutions for ϕ usually used in the context of the quintessential scenario. For the second choice, the theory allows three different solutions one of which is a scaling solution with equation of state $p_\phi = w\rho_\phi$ where w is predicted to be restricted by $-1 < w < -0.82$. The regime where the fermionic matter dominates (as compared to the dilatonic contribution) is analyzed. There it is found that starting from a single fermionic field we

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obtain exactly three different types of spin 1/2 particles in CEF that appears to suggest a new approach to the family problem of particle physics. It is automatically achieved that for two of them, fermion masses are constants, the energy-momentum tensor is canonical and the "fifth force" is absent. For the third type of particles, a fermionic self-interaction appears as a result of SSB of scale invariance.

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I. INTRODUCTION

Recent observations imply that the Universe now is undergoing era of acceleration [1]. This is most naturally explained by the existence of a vacuum energy which can be of the form of an explicit cosmological constant. Alternatively, there may be a slow rolling scalar field, whose potential (assumed to have zero asymptotic value) provides the negative pressure required for accelerating the Universe. This is the basic idea of the quintessence [2]. Some of the problems of the quintessence scenario connected to the field theoretic grounds of this idea, are: i) what is the origin of the quintessence potential; ii) why the asymptotic value of the potential vanishes (this is actually the "old" cosmological constant problem [4]); iii) the needed flatness of the potential [5]; iv) without the symmetry $\phi \to \phi + const$ it is very hard to explain the absence of the long-range force if no fine tuning is made [6,7], but such a translation-like symmetry is usually incompatible with a nontrivial potential.

One of the main aims of this paper is to show how the above problems can be solved in the context of the two measures theories (TMT) [8–15]. These kind of models are based on the observation that in a generally covariant formulation of the action principle one has to integrate using an invariant volume element, which is not obliged to be dependent of the metric. In GR, the volume element $\sqrt{-g}d^4x$ is indeed generally coordinate invariant, but nothing forbids us from considering the invariant volume element Φd^4x where Φ is a scalar density that could be independent of the metric [8].

If the measure Φ is allowed, we have seen in a number of models [9–12] that, in the conformal Einstein frame (CEF), the equations of motion have the canonical GR structure, but the scalar field potential produced in the CEF is such that zero vacuum energy for the ground state of the theory is obtained without fine tuning, that is the "old" cosmological constant problem can be solved [11].

If both measures ($\sqrt{-g}$ and Φ) are allowed, this opens new possibilities concerning scale invariance [12–15]. In this context we study here a theory which is invariant under scale transformations including also a translation-like symmetry for a dilaton field of the form

$\phi \to \phi + const$ discussed by Carroll [6]. For the case when the original action does not contain dilaton potentials at all, it is found that the integration of the equation of motion corresponding to the measure Φ degrees of freedom, spontaneously breaks the scale symmetry and the generation of a dilaton potential is a consequence of this spontaneous symmetry breaking (SSB). When studying the theory in the CEF, it is demonstrated in Sec. III that the spontaneously induced dilaton potential has the exponential form and in addition, also non-linear kinetic terms appear in general.

In Sec. IV we discuss possible cosmological applications of the theory when the dilaton field is the dominant fraction of the matter: it is found that quintessential solutions are possible.

In Sec. V we show that in the presence of fermions, the theory displays a successful fermionic mass generation after the spontaneous symmetries break (SSB), and this is actually the second main aim of this paper. In the regime when the fermionic density is of the order typical for the normal particle physics (which in the laboratory conditions is always much higher than the dilaton density), there are constant fermion masses, gravitational equations are canonical and the "fifth force" is absent, - all this without any additional restrictions on the parameters of the theory. A possible way for explanation to the "family puzzle" of particle physics also appears naturally in the context of this model. For one of the families, a fermion self-interaction appears as a result of the SSB of scale symmetry.

II. TWO MEASURES THEORY (TMT)

The main idea of these kind of theories [8–11] is to reconsider the basic structure of generally relativistic actions, which are usually taken to be of the form

$$S = \int d^4x \sqrt{-g} L \tag{1}$$

where L is a scalar and $g = \det(g_{\mu\nu})$. The volume element $d^4x\sqrt{-g}$ is an invariant entity. It is however possible to build a different invariant volume element if another density, that is an object having the same transformation properties as $\sqrt{-g}$, is introduced. For example, given four scalar fields φ_a, $a = 1, 2, 3, 4$ we can build the density

$$\Phi = \varepsilon^{\mu\nu\alpha\beta}\varepsilon_{abcd}\partial_\mu\varphi_a\partial_\nu\varphi_b\partial_\alpha\varphi_c\partial_\beta\varphi_d \tag{2}$$

and then Φd^4x is also an invariant object. Notice also that Φ is a total derivative since

$$\Phi = \partial_\mu(\varepsilon^{\mu\nu\alpha\beta}\varepsilon_{abcd}\varphi_a\partial_\nu\varphi_b\partial_\alpha\varphi_c\partial_\beta\varphi_d) \tag{3}$$

Therefore if we consider possible actions which use both Φ and $\sqrt{-g}$ we are lead to TMT

$$S = \int L_1\Phi d^4x + \int L_2\sqrt{-g}d^4x \tag{4}$$

Since Φ is a total derivative, a shift of L_1 by a constant, $L_1 \to L_1 + const$, has the effect of adding to S the integral of a total derivative , which does not change equations of motion. Such a feature is not present in the second piece of Eq. (4) since $\sqrt{-g}$ is not a total derivative. It is clear then that the introduction of a new volume element has consequences on the way we think about the cosmological constant problem, since the vacuum energy is related to the coupling of the volume element with the Lagrangian. How this relation is modified when a new volume element is introduced, was discussed in [9–11].

It has been shown that a wide class of TMT models [11], containing among others a scalar field, can be formulated which are free of the "old" cosmological constant problem. An important feature of those models consists in the use of the "first order formalism" where the connection coefficients $\Gamma^\lambda_{\mu\nu}$, metric $g_{\mu\nu}$ and in our case also φ_a and any matter fields that may exist are treated as independent dynamical variables. Any relations that they satisfy are a result of the equations of motion. The models allow the use of the so called conformal Einstein frame (CEF) where the equations of motion have canonical GR form and the effective potential has an absolute minimum at zero value of the effective energy density without fine tuning. This was verified to be the case in all examples studied in Ref. [11], provided the action form (4) is preserved, where L_1 and L_2 are φ_a-independent. If this is so, an infinite symmetry appears [11]: $\varphi_a \to \varphi_a + f_a(L_1)$, where $f_a(L_1)$ is an arbitrary function of L_1.

III. SCALE INVARIANT MODEL WITH SPONTANEOUS SYMMETRY BREAKING GIVING RISE TO A POTENTIAL

If we believe that there are no fundamental scales in physics, we are lead to the notion of scale invariance. In the context of TMT, to implement global scale invariance one has to introduce a "dilaton" field [12,13]. In this case the measure Φ degrees of freedom also can participate in the scale transformation [12,13]. In [12,13], explicit potentials (of exponential form) which respect the symmetry were introduced. Fundamental theories however, like string theories, etc. give most naturally only massless particles, which means that only kinetic terms and no explicit potentials appear from the beginning naturally. Let us therefore explore a similar situation in the context of a scale invariant TMT model. We postulate then the form of the action

$$S = \int d^4x \Phi e^{\alpha\phi/M_p} \left[-\frac{1}{\kappa}R(\Gamma,g) + \frac{1}{2}g^{\mu\nu}\phi_{,\mu}\phi_{,\nu}\right] + \int d^4x \sqrt{-g} e^{\alpha\phi/M_p} \left[-\frac{b_g}{\kappa}R(\Gamma,g) + \frac{b_k}{2}g^{\mu\nu}\phi_{,\mu}\phi_{,\nu}\right] \tag{5}$$

where we proceed in the first order formalism and $R(\Gamma,g) = g^{\mu\nu}R_{\mu\nu}(\Gamma)$, $\quad R_{\mu\nu}(\Gamma) = R^{\alpha}_{\mu\nu\alpha}(\Gamma)$ and $R^{\lambda}_{\mu\nu\sigma}(\Gamma) \equiv \Gamma^{\lambda}_{\mu\nu,\sigma} + \Gamma^{\lambda}_{\alpha\sigma}\Gamma^{\alpha}_{\mu\nu} - (\nu \leftrightarrow \sigma)$. By means of a redefinition of factors of ϕ and of Φ one can always normalize the kinetic term of ϕ and the R-term that go together with Φ as done in (5). Once this is done, this freedom however is not present any more concerning the second part of the action going together with $\sqrt{-g}$. The appearance of the constants b_g and b_k is a result of this.

The action (5) is invariant under the scale transformations:

$$\begin{aligned} &g_{\mu\nu} \rightarrow e^{\theta} g_{\mu\nu}, \quad \phi \rightarrow \phi - \frac{M_p}{\alpha}\theta, \quad \Gamma^{\sigma}_{\mu\nu} \rightarrow \Gamma^{\sigma}_{\mu\nu}, \\ &\varphi_a \rightarrow \lambda_a \varphi_a, \quad a = 1,2,3,4 \quad where \quad \Pi\lambda_a = e^{2\theta}. \end{aligned} \tag{6}$$

Notice that (5) is the most general action of TMT invariant under the scale transformations (6) where the Lagrangian densities L_1 and L_2 are linear in the scalar curvature and quadratic in the space-time derivatives of the dilaton but *without explicit potentials*. In Refs.

[12,13], actions of such type were discussed, but with explicit potentials and without kinetic term going with $\sqrt{-g}$. A different definition of the metric have been used also in [12,13] ($g^{\mu\nu}$ in [12,13] instead of the combination $e^{\alpha\phi/M_p}g^{\mu\nu}$ here) so that no factor $e^{\alpha\phi/M_p}$ appeared multiplying Φ in Ref. [12,13]. Also it is possible to formulate a consistent scale invariant model keeping only the simplest structure (namely, only the measure Φ is used), provided L_1 contains 4-index field strengths and an exponential potential for the dilaton [14]. Then SSB of the scale invariance can lead to a quintessential potential [14]. Another type of the field theory models with explicitly broken scale symmetry have been studied in Ref. [15] where it is shown that the quintessential inflation [16] type models can be obtained without fine tuning.

We examine now the equations of motion that arise from (5). Varying the measure fields φ_a, we get

$$A^{\mu}_{a}\partial_{\mu}[-\frac{1}{\kappa}R(\Gamma,g)+\frac{1}{2}g^{\alpha\beta}\phi_{,\alpha}\phi_{,\beta}]=0 \tag{7}$$

$$A^{\mu}_{a}=\varepsilon^{\mu\nu\alpha\beta}\varepsilon_{abcd}\partial_{\nu}\varphi_{b}\partial_{\alpha}\varphi_{c}\partial_{\beta}\varphi_{d}. \tag{8}$$

Since $Det(A^{\mu}_{a})=\frac{4^{-4}}{4!}\Phi^{3}$ it follows that if $\Phi\neq 0$,

$$-\frac{1}{\kappa}R(\Gamma,g)+\frac{1}{2}g^{\mu\nu}\phi_{,\mu}\phi_{,\nu}=sM^{4}=const, \tag{9}$$

where $s=\pm 1$ and M is a constant with the dimension of mass. It can be noticed that the appearance of a nonzero integration constant sM^4 spontaneously breaks the scale invariance (6).

The variation of S with respect to $g^{\mu\nu}$ yields

$$-\frac{1}{\kappa}R_{\mu\nu}(\Gamma)(\Phi+b_g\sqrt{-g})+\frac{1}{2}\phi_{,\mu}\phi_{,\nu}(\Phi+b_k\sqrt{-g})-\frac{1}{2}\sqrt{-g}g_{\mu\nu}\left[-\frac{b_g}{\kappa}R(\Gamma,g)+\frac{b_k}{2}g^{\alpha\beta}\phi_{,\alpha}\phi_{,\beta}\right]=0 \tag{10}$$

Contracting Eq. (10) with $g^{\mu\nu}$, solving for $R(\Gamma,g)$ and inserting into Eq. (9) we obtain the constraint

$$M^4(\zeta - b_g)e^{-\alpha\phi/M_p} + \frac{\Delta}{2} g^{\alpha\beta}\phi_{,\alpha}\phi_{,\beta} = 0, \tag{11}$$

where the scalar ζ is the ratio of two measures

$$\zeta \equiv \frac{\Phi}{\sqrt{-g}} \tag{12}$$

and $\Delta = b_g - b_k$. It is very interesting that *the geometrical quantity ζ is defined by a constraint where neither Newton constant nor curvature enter.*

Varying the action with respect to ϕ and using Eq. (9) we get

$$(-g)^{-1/2}\partial_\mu \left[(\zeta + b_k)e^{\alpha\phi/M_p}\sqrt{-g}g^{\mu\nu}\partial_\nu\phi)\right] - \frac{\alpha}{M_p}\left[M^4(\zeta + b_g) - \frac{\Delta}{2} g^{\alpha\beta}\phi_{,\alpha}\phi_{,\beta}e^{\alpha\phi/M_p}\right] = 0 \tag{13}$$

Considering the term containing connection $\Gamma^\lambda_{\mu\nu}$, that is $R(\Gamma, g)$, we see that it can be written as

$$S_\Gamma = -\frac{1}{\kappa}\int \sqrt{-g}e^{\alpha\phi/M_p}(\zeta + b_g)g^{\mu\nu}R_{\mu\nu}(\Gamma) = -\frac{1}{\kappa}\int \sqrt{-\tilde{g}}\tilde{g}^{\mu\nu}R_{\mu\nu}(\Gamma), \tag{14}$$

where $\tilde{g}_{\mu\nu}$ is determined by the conformal transformation

$$\tilde{g}_{\mu\nu} = e^{\alpha\phi/M_p}(\zeta + b_g)g_{\mu\nu} \tag{15}$$

It is clear then that the variation of S_Γ with respect to Γ will give the same result expressed in terms of $\tilde{g}_{\mu\nu}$ as in the similar GR problem in Palatini formulation. Therefore, if $\Gamma^\lambda_{\mu\nu}$ is taken to be symmetric in μ, ν, then in terms of the metric $\tilde{g}_{\mu\nu}$, the connection coefficients $\Gamma^\lambda_{\mu\nu}$ are Christoffel's connection coefficients of the Riemannian space-time with the metric $\tilde{g}_{\mu\nu}$:

$$\Gamma^\lambda_{\mu\nu} = \{^{\lambda}_{\mu\nu}\}|_{\tilde{g}_{\mu\nu}} = \frac{1}{2}\tilde{g}^{\lambda\alpha}(\partial_\nu\tilde{g}_{\alpha\mu} + \partial_\mu\tilde{g}_{\alpha\nu} - \partial_\alpha\tilde{g}_{\mu\nu}). \tag{16}$$

So, it appears that working with $\tilde{g}_{\mu\nu}$, we recover a Riemannian structure for space-time. We will refer to this as the conformal Einstein frame (CEF). Notice that $\tilde{g}_{\mu\nu}$ is invariant under the scale transformations (6) and therefore the spontaneous breaking of the global scale symmetry (see Eq. (9) and discussion after it) is reduced, in CEF, to the spontaneous breaking of the shift symmetry $\phi \to \phi + const$ for the dilaton field. In this context, it is interesting to notice that Carroll [6] pointed to the possible role of the shift symmetry for a scalar field in the resolution of the long range force problem of the quintessential scenario.

Equations (10) and (13) in CEF take the following form:

$$G_{\mu\nu}(\tilde{g}_{\alpha\beta}) = \frac{\kappa}{2} T^{eff}_{\mu\nu} \tag{17}$$

$$T^{eff}_{\mu\nu} = \frac{1}{2}\left(1 + \frac{b_k}{b_g}\right)(\phi_{,\mu}\phi_{,\nu} - K\tilde{g}_{\mu\nu}) - \frac{\Delta^2 K e^{2\alpha\phi/M_p}}{2b_g M^4}\left(\phi_{,\mu}\phi_{,\nu} - \frac{1}{2}K\tilde{g}_{\mu\nu}\right) + \tilde{g}_{\mu\nu}\frac{sM^4}{4b_g}e^{-2\alpha\phi/M_p} \tag{18}$$

$$N\left[(-\tilde{g})^{-1/2}\partial_\mu(\sqrt{-\tilde{g}}\tilde{g}^{\mu\nu}\partial_\nu\phi) + \tilde{g}^{\alpha\beta}\partial_\alpha\phi\partial_\beta \ln N\right] + \frac{\alpha\Delta^2}{M_p M^4}K^2 e^{2\alpha\phi/M_p} - \frac{\alpha M^4}{M_p}e^{-2\alpha\phi/M_p} = 0 \tag{19}$$

Here

$$K \equiv \frac{1}{2}\tilde{g}^{\alpha\beta}\phi_{,\alpha}\phi_{,\beta}, \quad N \equiv b_g + b_k - \frac{\Delta^2}{M^4}K e^{2\alpha\phi/M_p}, \tag{20}$$

$G_{\mu\nu}(\tilde{g}_{\alpha\beta})$ is the Einstein tensor in the Riemannian space-time with metric $\tilde{g}_{\mu\nu}$ and the constraint (11) have been used which in CEF takes the form

$$\zeta = b_g \frac{M^4 - \Delta K e^{2\alpha\phi/M_p}}{M^4 + \Delta K e^{2\alpha\phi/M_p}} \tag{21}$$

Notice that in $T^{eff}_{\mu\nu}$ we can recognize an effective potential

$$V_{eff} = \frac{sM^4}{4b_g}e^{-2\alpha\phi/M_p} \tag{22}$$

which appears in spite of the fact that no explicit potential term was introduced in the original action (5). As we see, the existence of V_{eff} is associated with the constant sM^4, appearance of which spontaneously breaks the scale invariance. This is actually a new mechanism for generating the exponential potential[1].

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[1]See for comparison Refs. [17–19] and a general discussion in Ref. [3]

Notice also that if $b_g \neq b_k$, the effective energy-momentum $T^{eff}_{\mu\nu}$ as well as the dilaton equation of motion contain the non-canonical terms nonlinear[2] in gradients of the dilaton ϕ. It will be very important that the non-canonical in $\phi_{,\alpha}$ terms are multiplied by a very specific exponential of ϕ. As we will see, these non-canonical terms may be responsible for the most interesting scaling solutions. In the context of FRW cosmology, this structure provides conditions for quintessential solutions if $s = 1$.

IV. SCALING SOLUTIONS

In the context of a spatially flat FRW cosmology with a metric $ds^2_{eff} = \tilde{g}_{\mu\nu}dx^\mu dx^\nu = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2)$, the equations (17)-(19), with the choice $s = +1$, become:

$$H^2 = \frac{1}{3M_p^2}\rho_{eff}(\phi) \tag{23}$$

$$\begin{aligned}&\left(b_g + b_k - \frac{\Delta^2}{2M^4}\dot{\phi}^2 e^{2\alpha\phi/M_p}\right)\left[\ddot{\phi} + 3H\dot{\phi} + \dot{\phi}\partial_t \ln\left|b_g + b_k - \frac{\Delta^2}{2M^4}\dot{\phi}^2 e^{2\alpha\phi/M_p}\right|\right]\\ &+\frac{\alpha\Delta^2}{4M^4M_p}\dot{\phi}^4 e^{2\alpha\phi/M_p} - \frac{\alpha M^4}{M_p}e^{-2\alpha\phi/M_p} = 0\end{aligned} \tag{24}$$

where the energy density of the dilaton field is

$$\rho_{eff}(\phi) = \frac{1}{4}\left(1 + \frac{b_k}{b_g}\right)\dot{\phi}^2 - \frac{3\Delta^2}{16b_gM^4}\dot{\phi}^4 e^{2\alpha\phi/M_p} + \frac{M^4}{4b_g}e^{-2\alpha\phi/M_p} \tag{25}$$

and the pressure

$$p_{eff}(\phi) = \frac{1}{4}\left(1 + \frac{b_k}{b_g}\right)\dot{\phi}^2 - \frac{\Delta^2}{16b_gM^4}\dot{\phi}^4 e^{2\alpha\phi/M_p} - \frac{M^4}{4b_g}e^{-2\alpha\phi/M_p} \tag{26}$$

One can see that Eqs. (23)-(25) allow solutions of a familiar quintessential form [2,3]

$$\phi(t) = \frac{M_p}{2\alpha}\phi_0 + \frac{M_p}{\alpha}\ln(M_p t) \tag{27}$$

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[2]Other possible origin for the non-linear kinetic terms, known in the literature [20], are higher order gravitational corrections in string and supergravity theories.

$$a(t) = t^{\gamma} \tag{28}$$

which provides scaling behaviors of the dilaton energy density

$$\rho_{eff}(\phi) \propto 1/a^{n}. \tag{29}$$

The important role for possibility of such solutions belongs to the remarkable feature of the nonlinear terms in Eqs. (23)-(25) that appear only in the combination $\dot{\phi}^2 e^{2\alpha\phi/M_p}$ which remains constant for the solutions (27) and (28):

$$\dot{\phi}^2 e^{2\alpha\phi/M_p} = const \tag{30}$$

Eqs. (27)-(29) describe solutions of Eqs. (23)-(25) with $n = \frac{2}{\gamma}$ if

$$\gamma = \frac{b_g + b_k - y}{4b_g\alpha^2} \tag{31}$$

where

$$y \equiv \frac{\Delta^2 M_p^4 e^{\phi_0}}{2M^4\alpha^2} \tag{32}$$

is a solution of the cubic equation

$$y^3 - 2(b_g + b_k - b_g\alpha^2)y^2 + (b_g + b_k)(b_g + b_k - \frac{4}{3}b_g\alpha^2)y - \frac{2}{3}b_g\alpha^2\Delta^2 = 0. \tag{33}$$

Up to now we did not make any assumptions about parameters of the theory. We will now suppose that b_g and b_k are positive and consider two particular cases.

The case I. If

$$b_k = b_g = b \tag{34}$$

then one can immediately see that Eqs. (23)-(26) describe the FRW cosmological model in the context of the standard GR when the minimally coupled scalar field ϕ with the potential $\frac{M^4}{4b}e^{-2\alpha\phi/M_p}$ is the only source of gravity. In this case the scaling solution (27), (28) coincides with the standard one [3] where

$$\gamma = \frac{1}{2\alpha^2}, \qquad n = 4\alpha^2. \tag{35}$$

The case II. Another interesting possibility consists of the assumption that

$$b_k \ll b_g \tag{36}$$

Then ignoring corrections of the order of b_k/b_g, the solutions of Eq. (33) are

$$y_1 = b_g \tag{37}$$

$$y_2 = \frac{b_g}{2}\left[1 - 2\alpha^2 + \sqrt{4\alpha^4 - \frac{20}{3}\alpha^2 + 1}\right] \tag{38}$$

$$y_3 = \frac{b_g}{2}\left[1 - 2\alpha^2 - \sqrt{4\alpha^4 - \frac{20}{3}\alpha^2 + 1}\right] \tag{39}$$

The solution y_1 corresponds to the static universe ($\gamma = 0$ and $a(t) = const$) supported by the slow rolling scalar field ϕ, Eq. (27). However, taking into account corrections of the order b_k/b_g to y_1 we will get $\gamma \propto \mathcal{O}(b_k/b_g)$.

Solutions y_2 and y_3 exist and are positive (see the definition (32)) only if

$$\alpha^2 \leq \frac{1}{6} \tag{40}$$

The solution y_2 corresponds to the values of the parameter γ monotonically varying from $\gamma_{min} = 2/3$ up to $\gamma = 1$ as α^2 changes from 0 up to 1/6.

The most interesting solution is given by y_3 that provides the values of the parameter γ monotonically varying from $\gamma_{min} = 1$ up to ∞ as α^2 changes from 1/6 up to zero. In this case, Eqs. (27)-(28) describe an accelerated universe for all permissible values of α^2 and the energy density of the dilaton field scales as in Eq. (29) with monotonically varying n, $2 \geq n \geq 0$ as α^2 changes from 1/6 up to zero. For the dilatonic matter equation-of-state $p = w\rho$ we get

$$-1 \leq w \leq -32/39 \approx -0.82 \tag{41}$$

In the conclusion of this section let us revert to one of the problems of the quintessence discussed in Introduction, namely to the flatness problem [5]. This is a question of the field theoretic basis for the choice of the flat enough potential. In fact, Kolda and Lyth noted [5] that an extreme fine tuning is needed in order to prevent the contribution from another possible terms breaking the flatness of the potential (see also for a review by Binetruy in Ref. [4]). In the theory we study here, there is a symmetry (scale symmetry (6)) which forbids the appearance of such dangerous contributions into V_{eff}, at least on the classical level. One can hope that the soft breaking of the scale symmetry guaranties that the symmetry breaking quantum corrections to the classical effective potential (22) will be small.

Here we have to make a note concerning quantization of the dilaton field. If $\Delta \neq 0$ then one can see from Eq. (25) that there is a possibility of negative energy contribution from the space-time derivatives of the dilaton. This raises of course the suspicion that the quantum theory may contain ghosts. Let us see that this problem does not appear when considering small perturbations around the background determined by the studied above scaling solutions. To see this, let us calculate the canonically conjugate momenta to ϕ, starting from the original action (5) and expressing it in terms of the variables defined in CEF, Eq. (15):

$$\pi_\phi = \frac{1}{2b_g}\left(b_g + b_k - \frac{\Delta^2}{sM^4}Ke^{2\alpha\phi/M_p}\right)\sqrt{-\tilde{g}}\tilde{g}^{00}\dot{\phi} \tag{42}$$

As we have seen, the cosmological scaling solutions provide backgrounds where $Ke^{2\alpha\phi/M_p} = const.$ Moreover, it is easy to see that for the scaling solutions

$$\pi_\phi = \frac{1}{2b_g}\left(b_g + b_k - y\right)a^3\dot{\phi} = 2\alpha^2\gamma a^3\dot{\phi}, \tag{43}$$

where γ and y are defined by Eqs. (31) and (32). We have seen also that for studied scaling solutions, γ gets positive values. Therefore we conclude that in such backgrounds π_ϕ and $\dot{\phi}$ have the same sign, that guaranties a ghost-free quantization. The only exclusion is the particular case when $b_k = 0$, $y = b_g$. As we have seen, such solution describes a static universe. In this case the canonically conjugate momenta $\pi_\phi = 0$ and therefore it appears that in this vacuum there are no particles associated with the scalar field ϕ.

V. SCALE INVARIANT FERMION-DILATON COUPLING WITHOUT THE LONG-RANGE FORCE PROBLEM

In general scalar-tensor theories, particle masses depend on time, when the theory is studied in the frame where Newton's constant is really a constant. However, for all the fermionic matter observed in the universe, the cosmological variation of particle masses (including those of electrons) is highly constrained. We want to show now how the theory presented in this paper avoids this problem and also the so called fifth force problem, in spite of the need to include exponential couplings of the dilaton field to fermionic matter in order to ensure global scale invariance.

To describe fermions, normally one uses the vierbein (e^{μ}_{a}) and spin-connection (ω^{ab}_{μ}) formalism where the metric is given by $g^{\mu\nu} = e^{\mu}_{a} e^{\nu}_{b} \eta^{ab}$ and the scalar curvature is $R(\omega, e) = e^{a\mu} e^{b\nu} R_{\mu\nu ab}(\omega)$ where

$$R_{\mu\nu ab}(\omega) = \partial_{\mu}\omega_{\nu ab} + \omega^{c}_{\mu a}\omega_{\nu cb} - (\mu \leftrightarrow \nu). \tag{44}$$

Following the general idea of the model, we now treat the geometrical objects e^{μ}_{a}, ω^{ab}_{μ}, the measure fields φ_a, as well as the dilaton ϕ and the fermionic fields as independent variables. In this formalism, the natural generalization of the action (5) keeping the general structure (4), when a fermion field Ψ is also present and which also respect scale invariance is the following:

$$\begin{aligned} S = & \int d^4x e^{\alpha\phi/M_p}(\Phi + b\sqrt{-g}) \left[-\frac{1}{\kappa} R(\omega, e) + \frac{1}{2} g^{\mu\nu}\phi_{,\mu}\phi_{,\nu} \right] \\ + & \int d^4x e^{\alpha\phi/M_p} \left[(\Phi + k\sqrt{-g}) \frac{i}{2} \overline{\Psi} \left(\gamma^a e^{\mu}_{a} \overrightarrow{\nabla}_{\mu} - \overleftarrow{\nabla}_{\mu} \gamma^a e^{\mu}_{a} \right) \Psi - (\Phi + h\sqrt{-g}) e^{\frac{1}{2}\alpha\phi/M_p} m \overline{\Psi}\Psi \right] \end{aligned} \tag{45}$$

where $\overrightarrow{\nabla}_{\mu} = \overrightarrow{\partial}_{\mu} + \frac{1}{2}\omega^{cd}_{\mu}\sigma_{cd}$ and $\overleftarrow{\nabla}_{\mu} = \overleftarrow{\partial}_{\mu} - \frac{1}{2}\omega^{cd}_{\mu}\sigma_{cd}$.

The action (45) is invariant under the global scale transformations

$$\begin{aligned} & e^{a}_{\mu} \to e^{\theta/2} e^{a}_{\mu}, \quad \omega^{\mu}_{ab} \to \omega^{\mu}_{ab}, \quad \varphi_a \to \lambda_a \varphi_a \quad where \quad \Pi\lambda_a = e^{2\theta} \\ & \phi \to \phi - \frac{M_p}{\alpha}\theta, \quad \Psi \to e^{-\theta/4}\Psi, \quad \overline{\Psi} \to e^{-\theta/4}\overline{\Psi}. \end{aligned} \tag{46}$$

In (45) two types of fermionic "kinetic-like terms" (as well as "mass-like terms") which respect scale invariance have been introduced: they are coupled to the measure Φ and to the measure $\sqrt{-g}$ respectively. As we have discussed in the previous section, the quantum theory may in general contain ghosts if $b_g \neq b_k$. Taking this into account and also for the sake of a simplification of the presentation of the results we have chosen $b_g = b_k = b$. Notice however that in the framework of the classical theory, all conclusions will be made below are true also if $b_g \neq b_k$. Except for this, Eq.(45) describes the most general action[3] satisfying the formulated above symmetries.

We can immediately obtain the equations of motion. From these going through similar steps to those performed in Sec. III, a constraint follows again which replaces (11) and which contains now a contribution from the fermions. The spin-connection can be found by the variation of ω^{μ}_{ab}.

Similar to what we learned from the treatment of Sec.III, we can consider the theory in the CEF which in this case involves also a transformation of the fermionic fields:

$$\tilde{g}_{\mu\nu} = e^{\alpha\phi/M_p}(\zeta + b)g_{\mu\nu}, \quad \tilde{e}_{a\mu} = e^{\frac{1}{2}\alpha\phi/M_p}(\zeta + b)^{1/2}e_{a\mu},$$
$$\Psi' = e^{-\frac{1}{4}\alpha\phi/M_p}\frac{(\zeta + k)^{1/2}}{(\zeta + b)^{3/4}}\Psi \tag{47}$$

In terms of these variables, the transformed spin-connections $\tilde{\omega}^{cd}_{\mu}$ turns out to be that of the Einstein-Cartan space-time and, besides, the new variables $\tilde{g}_{\mu\nu}$, $\tilde{e}_{a\mu}$, Ψ' and $\overline{\Psi}'$ are invariant under the scale transformations (46). In the CEF the only field which still has a non trivial transformation property is the dilaton ϕ which gets shifted (according to (46)). Thus, the presence of fermions does not change a conclusion made in Sec.III after Eq.(16): the spontaneous breaking of the scale symmetry is reduced, in the CEF, to the spontaneous breaking of the shift symmetry $\phi \rightarrow \phi + const$ for the dilaton field.

In terms of $\tilde{e}_{a\mu}$, Ψ', $\overline{\Psi}'$ and ϕ, the constraint (again arising as a self-consistency condition

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[3] Recall that in this paper we restrict ourselves to the models without explicit dilaton potentials in the original action

of equations of motion) which now replaces (21) and which contains now a contribution from the fermions is

$$(\zeta - b)M^4 e^{-2\alpha\phi/M_p} + F(\zeta)(\zeta + b)^2 m\overline{\Psi}'\Psi' = 0. \tag{48}$$

where we have chosen $s = +1$ for definiteness and the function $F(\zeta)$ is defined by

$$F(\zeta) \equiv \frac{1}{2(\zeta + k)^2(\zeta + b)^{1/2}}[\zeta^2 + (3h - k)\zeta + 2b(h - k) + kh] \tag{49}$$

The dilaton field equation is

$$(-\tilde{g})^{-1/2}\partial_\mu\left(\sqrt{-\tilde{g}}\tilde{g}^{\mu\nu}\partial_\nu\phi\right) - \frac{\alpha M^4}{M_p(\zeta + b)}e^{-2\alpha\phi/M_p} + \frac{\alpha m}{M_p}F(\zeta)\overline{\Psi}'\Psi' = 0. \tag{50}$$

The fermionic equation of motion in terms of the variables (47) takes the standard structure of that in the Einstein-Cartan space-time [21] where a fermion field is the only source of a non-riemannian part of the connection. The only novelty of the fermionic equation consists of the form of the ζ- depending fermion "mass" $m^{(eff)}(\zeta)$:

$$m^{(eff)}(\zeta) = \frac{m(\zeta + h)}{(\zeta + k)(\zeta + b)^{1/2}} \tag{51}$$

The gravitational equations are of the standard form (17) with

$$T^{eff}_{\mu\nu} = \phi_{,\mu}\phi_{,\nu} - K\tilde{g}_{\mu\nu} + \frac{b_g M^4}{(\zeta + b)^2}e^{-2\alpha\phi/M_p}\tilde{g}_{\mu\nu} + T^{(f,canonical)}_{\mu\nu} - mF(\zeta)\overline{\Psi}'\Psi'\tilde{g}_{\mu\nu}, \tag{52}$$

where

$$T^{(f,canonical)}_{\mu\nu} = \frac{i}{2}[\overline{\Psi}'\gamma^a e'_{a(\mu}\nabla_{\nu)}\Psi' - (\nabla_{(\mu}\overline{\Psi}')\gamma^a e'_{\nu)a}\Psi'] \tag{53}$$

is the canonical energy-momentum tensor for the fermionic field in the curved space-time [22] and $\nabla_\mu\Psi' = \left(\partial_\mu + \frac{1}{2}\tilde{\omega}^{cd}_\mu\sigma_{cd}\right)\Psi'$ and $\nabla_\mu\overline{\Psi}' = \partial_\mu\overline{\Psi}' - \frac{1}{2}\tilde{\omega}^{cd}_\mu\overline{\Psi}'\sigma_{cd}$.

The scalar field ζ is defined by the constraint (48) in terms of the dilaton and fermion fields as a solution of the seventh degree algebraic equation that makes finding ζ in general a very complicated question. However there are two physically most interesting limiting cases when solving (48) is simple enough.

Let us first analyze the constraint (48) when the fermionic density (proportional to $\overline{\Psi}'\Psi'$) is very low as compared to the contributions of the dilaton potential ($\propto M^4 e^{-2\alpha\phi/M_p}$). In this limiting case, the constraint gives again the expression (21) for ζ where we have to take now $\Delta = 0$, that is constraint yields the constant value[4] $\zeta = b$. Inserting this value of ζ into (51) we see that the mass of a "test" fermion (that is when we ignore the effect of the fermion itself on the dilatonic background) is constant.

An opposite regime is realized when the contribution of the fermionic density to the constraint (48) is very high as compared to the contribution of the dilaton potential. In the context of the present day universe, this regime corresponds in particular to the normal laboratory conditions in particle physics. Then according to the constraint (48), one of the possibilities for this to be realized consists in the condition

$$F(\zeta) = 0 \tag{54}$$

from which we find two possible *constant* values for ζ

$$\zeta_{1,2} = \frac{1}{2}\left[k - 3h \pm \sqrt{(k-3h)^2 + 8b(k-h) - 4kh}\,\right] \tag{55}$$

These solutions, i.e. values ζ_1 and ζ_2, are real and different for very broad range of the parameters b, k and h. These conditions have to be considered together with the obvious requirement $\zeta + b > 0$ (see transformations (47)). For instance, for $h > 0$, all these conditions are satisfied provided that parameters are situated in the broad region defined by the system of inequalities $(b-h)(b-k) > 0$ and $(k-h)[k-h+8(b-h)] > 0$.

We see from (51) that two different constants ζ given by (55) define in general *two specific masses* for the fermion. We will assume that these two fermionic states should be identified with the first two fermionic generations.

The separate possibility relevant to the high fermionic density (again, as compared to the contributions of the dilaton potential) is the case when

[4]Notice that if we had chosen $b_g \neq b_k$ and assumed the quintessential cosmological solution of Sec.IV where $Ke^{2\alpha\phi/M_p} = const$, we would again get a constant value of ζ.

$$\zeta + b \approx 0 \tag{56}$$

is a solution. However, the solution $\zeta + b = 0$ is singular one as we see from equations of motion. This means that one can not neglect the first term in the constraint (48) and instead of $\zeta + b = 0$ we have to take the solution $\zeta_3 \approx -b$ by solving $\zeta + b$ in terms of the dilaton field and the primordial fermion field itself. Then it follows from (48) and (49) that

$$\frac{1}{\sqrt{\zeta_3 + b}} \approx \left[\frac{m(h-b)}{4M^4 b(k-b)}\overline{\Psi}'\Psi' e^{2\alpha\phi/M_p}\right]^{1/3}. \tag{57}$$

Therefore, instead of constant masses, as it was for ζ_1 and ζ_2 (i.e. in the case $F(\zeta) = 0$), this leads to higher fermion self-interaction which can be represented by the following term in the effective fermion Lagrangian in the dilatonic background $\phi = \bar{\phi}$:

$$L^{ferm}_{selfint} = 3\left[\frac{1}{b}\left(\frac{m(h-b)}{4M(k-b)}\overline{\Psi}'\Psi'\right)^4 e^{2\alpha\bar{\phi}/M_p}\right]^{1/3}. \tag{58}$$

The coupling constant of this self-interaction depends on the dilaton ϕ. The condition (56) is realized, for example, as the classical cosmological background value $\phi = \bar{\phi}(t) \to \infty$ that corresponds to the late universe in the quintessence scenario. A full treatment of the case with $\zeta = \zeta_3$, which we assume corresponds to the third fermion generation, requires the study of quantum corrections. We expect that after $\overline{\Psi}'\Psi'$ develops an expectation value, the fermion condensate will give the third family appropriate masses similar to what we know in NJL model [23] (for recent progress in this subject see e. g. Ref. [24]). It is interesting to note that appearance of the higher fermion self-interaction here is related to the SSB of the scale invariance. In fact, the appearance of the integration constant M in Eqs. (57) and (58) tells us that without SSB of scale invariance such interaction is not defined.

Concluding this analysis of equations when the fermionic density is of the order typical for the normal particle physics (which in the laboratory conditions is always much higher than the dilaton density) we see that starting from a single primordial fermionic field we obtain exactly three different types of spin 1/2 particles in the CEF. This appears to be a new approach to the family problem in particle physics and it will be subject of a detail study in another publication.

Coming back to the first two fermion families generated in the regime of fermion dominance as $F(\zeta) = 0$ we note that surprisingly the same factor $F(\zeta)$ appears in the last terms of Eqs. (50) and (52). Therefore, in the regime where *regular* fermionic matter (i.e. u and d quarks, e^- and ν_e) is a dominant fraction, the last terms of Eqs. (50) and (52) *automatically* vanish. In Eq. (50), this means that the fermion density $\overline{\Psi}'\Psi'$ is not a source for the dilaton and thus the long-range force disappears automatically. Notice that there is no need to require no interactions of the dilaton with regular matter at all to have agreement with observations but it is rather enough that these interactions vanish in the regime where regular fermionic matter dominates over other matter fields. In Eq. (52), the condition (54) means that in the region where the regular fermionic matter dominates, the fermion energy-momentum tensor becomes equal to the canonical energy-momentum tensor of a fermion field in GR. [5]

VI. DISCUSSION AND CONCLUSIONS

In this paper the possibility of a spontaneously generating exponential potential for the dilaton field in the context of TMT with spontaneously broken global scale symmetry was studied. The symmetry transformations formulated in terms of the original variables (6) (or (46) in the presence of fermions) include the global scale transformations of the metric, of the scalar fields φ_a related to the measure Φ (and of the fermion fields) and in addition the dilaton field ϕ undergoes a global shift. In the CEF (see Eqs. (15) or (47) where the theory is formulated in the Riemannian (or Einstein-Cartan) space-time), all dynamical variables

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[5] The decoupling of the dilaton in the CEF in the case of high fermion density was discussed also in a simpler spontaneously broken scale invariant model (with $b = k = 0$ and explicit exponential potentials) in Ref. [13]. In the framework of other TMT model [15] (with small *explicit* breaking of the scale invariance) in the context of the quintessential scenario, a special tuning of the parameters is needed to achieve the dilaton-fermion decoupling.

are invariant under the transformations (6) (or (46)) except for the dilaton field which still gets shifted by a constant. Thus, SSB of the scale symmetry that appears firstly in (9) when solving Eq. (7), is reduced, in the CEF, to SSB of the shift symmetry $\phi \rightarrow \phi + const.$

The original action does not includes potentials but in the CEF, the exponential potential appears as a result of SSB of the scale symmetry. In the generic case $\Delta = b_g - b_k \neq 0$, the process of SSB also produces terms with higher powers in derivatives of the dilaton field.

Cosmological scaling solutions of the theory were studied. The flatness of the potential V_{eff} which is associated here with the exponential form, is protected by the scale symmetry. Quintessence solutions (corresponding to accelerating universe) were found possible for a broad range of parameters.

Finally, the behavior of fermions in such type of models was investigated. Scale invariant fermion mass-like terms can be introduced in two different ways since they can appear coupled to each of the two different measures of the theory. Although an exponential of the dilaton field ϕ couples to the fermion in both of these terms, it is found that when the fermions are treated as a test particles in the scaling background, their masses in the CEF are constants.

Even more surprising is the behavior of the fermions in the limit of high fermion density as compared to the dilaton density. This approximation is regarded as more realistic if we are interested in the regular particle physics behavior of these fermions under normal laboratory conditions. It is found then that in the CEF, a given fermion can behave in three different ways according to the three different solutions of the fundamental constraint (48). Two of the solutions correspond to fermions with constant masses and the other - to a higher fermion self-interaction which, we expect, can generate mass on the quantum level in a manner similar to a NJL model [23]. From one primordial fermion three are obtained for free. This suggests a new approach to the "family problem" in particle physics.

In addition to this, for the two mentioned above solutions (55) corresponding to constant fermion masses, the fermion-dilaton coupling in the CEF (proportional to $F(\zeta)$, Eq.(49)) disappears automatically. If one of these types of fermions is associated to the first family (regular matter, i.e., u and d quarks, e^- and ν_e), we obtain that normal matter decouples from the dilaton.

All what has been done here concerning fermions is in the context of a toy model without Higgs fields, gauge bosons and the associated $SU(2) \times U(1) \times SU(3)$ gauge symmetry of the standard model. As we have seen in other models (see [11], the second reference of [13] and [15]), it is possible to incorporate the two measure ideas with the gauge symmetry and Higgs mechanism. Now the differences consist of: i) the presence of global scale symmetry, ii) the most general TMT structure for gravitation and dilaton sector. The complete discussion of the standard model in the context of such TMT structure will be presented in a separate publication [25]. Here we want only to explain shortly the main ideas that provides us the possibility to implement this program. It is important that in a simple way gauge fields can be incorporated so that they will not appear in the fundamental constraint[6] in contrast to the fermions (see for comparison Eq. (48)). We can also work without significant changes in the discussion of the fermionic sector if instead of explicit mass-like terms we will work with similar terms where the coupling constants with the dimensionality of the mass are replaced by gauge invariant Yukawa couplings to the Higgs field. Proceeding in the spontaneously broken $SU(2) \times U(1)$ gauge theory [25] and starting from one correspondent primordial fermions family we observe again [25] the effect of generation of three fermion families, as was above in the toy model. Generating mass of two of them is automatic as in the previous discussion. For the third we need again some quantum effect that gives rise to a fermion condensate.

The analysis of the constraint (48) provides in general seven solutions for ζ. It could be that among of them there is a solution corresponding to a fermionic state responsible for dark matter. For example, the solution (57) after inserting into 00-component of the

[6]This may be done by making the gauge field kinetic terms coupled only to $\sqrt{-g}$ which is dictated by local scale invariance of that part of the action.

energy-momentum tensor (52) makes the last three terms of (52) to be dependent in the same manner only on the combination $\overline{\Psi}'\Psi' e^{\frac{1}{2}\alpha\phi/M_p}$ and they appear to be of the same order of magnitude. This implies that fermion contributions to the energy and those of the scalar field are of the same order that provides then a possible explanation of the "cosmic coincidence" problem. A consistent study of these cosmological questions will become possible after we will explore in detail [25] the field theoretic aspects of the displayed here "families birth effect".

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